

# Numerical Techniques:

## Euler's Method

Patrick Marsh

Mid-America Christian University

24 February 2011

# Background

Let's begin with an initial value problem:

$$\frac{dy}{dt} = f(t, y); \quad y(t_0) = y_0$$

Since we are given  $f(t, y)$  we can plot the slope field. The idea behind Euler's Method is to start at point  $(t_0, y_0)$  in the slope field and take tiny steps dictated by the tangents in the slope field.

# Background

The slope of the approximate solution is updated every  $\Delta t$  units of  $t$ . In other words, for each step we move  $\Delta t$  units along the  $t$ -axis.

The size of  $\Delta t$  determines the accuracy of the approximate solution.

smaller  $\Delta t \implies$  more precise solution.

# The Algorithm: Euler's Method for $\frac{dy}{dt} = f(t, y)$

Given the initial condition  $y(t_0) = y_0$  and the step size of  $\Delta t$ , compute the point  $(t_{k+1}, y_{k+1})$  from the preceding point  $(t_k, y_k)$  as follows:

- 1 Use the ODE to compute the slope  $f(t_k, y_k)$
- 2 Calculate the next point  $f(t_{k+1}, y_{k+1})$  using the formulas:

$$t_{k+1} = t_k + \Delta t$$

and

$$y_{k+1} = y_k + f(t_k, y_k)\Delta t$$

## Approximating an Autonomous Equation

Consider  $\frac{dy}{dt} = 2y - 1$ ;  $y(0) = 1$ . Let's use a  $\Delta t = 0.1$  to approximate the solution over the domain  $0 \leq t_k \leq 1$ .

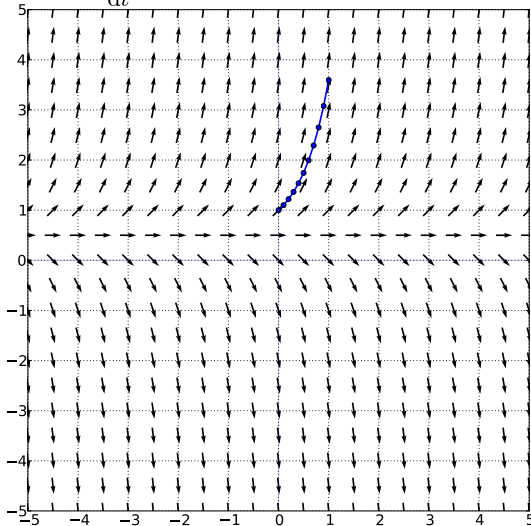
Thus, in this problem  $f(t, y) = 2y - 1$

$$\frac{dy}{dt} = 2y - 1; \quad y(0) = 1 \quad \Delta t = 0.1$$

$k$	$t_k$	$y_k$	$f(t_k, y_k)$
0	0	1	1
1	0.1	1.1	1.2
2	0.2	1.2	1.44
3	0.3	1.364	1.73
4	0.4	1.537	2.07
5	0.5	1.744	2.49
6	0.6	1.993	2.98
7	0.7	2.292	3.58
8	0.8	2.650	4.30
9	0.9	3.080	5.16
10	1.0	3.596	

# Graph

$$\frac{dy}{dt} = 2 * y - 1; \quad \Delta t = 0.1$$



## Approximating a Non-Autonomous Equation

Consider  $\frac{dy}{dt} = -2ty^2$ ;  $y(0) = 1$ . Let's use a  $\Delta t = 0.5$  to approximate the solution over the domain  $0 \leq t_k \leq 2$ .

Thus, in this problem  $f(t, y) = -2ty^2$

$$\frac{dy}{dt} = -2ty^2; \quad y(0) = 1. \text{ Let's use a } \Delta t = 0.5$$

$k$	$t_k$	$y_k$	$f(t_k, y_k)$
0	0	1	0
1	$\frac{1}{2}$	1	-1
2	0.2	$\frac{1}{2}$	$-\frac{1}{2}$
3	$\frac{3}{2}$	$\frac{1}{4}$	$-\frac{3}{16}$
4	3	$\frac{5}{32}$	

# Graph

$$\frac{dy}{dt} = -2ty^2 ; \quad \Delta t = 0.5$$

