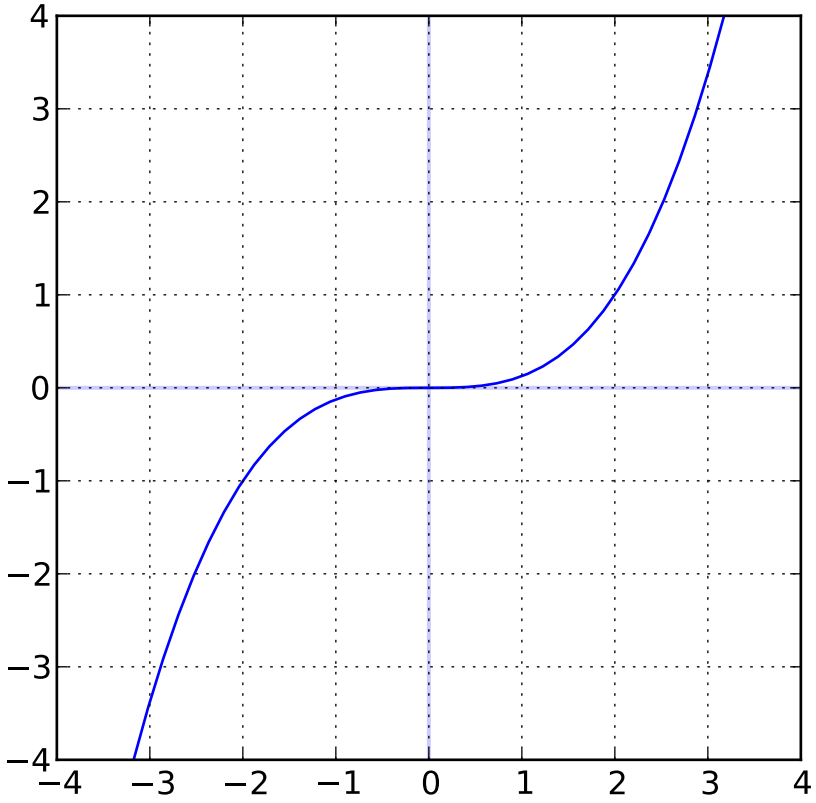
DUE: 31 March 2011

Please answer these questions on separate pieces of paper.
This Exam is open-book, open-note, open-Internet.
You cannot ask another person for help.
To receive full credit, you must show your work!!!

1. Short Answer Questions:
   (i) What is a differential equation?
   (ii) What is meant by the term "first-order differential equation"?
   (iii) What is meant by the term "separable differential equation"?
   (iv) What is an "autonomous differential equation"?
   (v) What is a "linear differential equation"?
   (vi) How can you tell if a differential equation is autonomous by looking at the slope field?
   (vii) What is the difference between a "homogeneous" and a "nonhomogeneous" differential equation?
   (viii) Give an example of a first-order, autonomous, linear, homogeneous differential equation.
   (ix) What is the general solution to your differential equation provided in the above question?
   (x) Has Grant fulfilled his debt to the class by bringing doughnuts?

2. Consider the following differential equation of the form $\frac{dy}{dt} = f(y)$, graphed below. Give a rough sketch of the slope field that corresponds to this equation. Note, please answer on the graph above, in pencil, plotting the slope field at each intersection of the dotted lines.



3. Consider the initial-value problem differential equation:

$$\frac{dy}{dt} = -2y + 2\ sin(2\pi t); \quad y(0) = 1$$

Using Euler's method, compute three different approximate solutions corresponding to $\triangle t = 1.0, 0.5$ over the interval $0 \leq t \leq 4$. Graph all three solutions. What predictions do you make about the actual solution to the initial-value problem?

4. In the following questions
   (a.) specify if the given equation is autonomous, linear, and/or homogeneous
   (b.) solve the initial value problem

(i) $\frac{dy}{dt} = -2ty; \quad y(0) = e$

(ii) $\frac{dy}{dt} = 2y + cos4t; \quad y(0) = 1$

(iii) $\frac{dy}{dt} = t^2y^3 + y^3; \quad y(0) = -1/2$

(iv) $\frac{dy}{dt} = 2ty + 3te^{t^2}; \quad y(0) = 1$

(v) $\frac{dy}{dt} = 1 - y^2; \quad y(0) = 1$

Bonus (*You Must Attempt!*)

The air in a small rectangular room 20 ft by 5 ft by 10 ft is 3% carbon monoxide. Starting at $t = 0$, air containing 1% carbon monoxide is blown into the room at the rate of 100 ft^3 per hour and well mixed air flows out through a vent at the same rate.

(i) Write an initial-value problem for the amount of carbon monoxide in the room over time.

(ii) When will the air in the room be 2% carbon monoxide?