DUE: 11 May 2011 by 4 PM CDT

Please answer these questions on separate pieces of paper.
This Exam is open-book, open-note, open-Internet.
You cannot ask another person for help.
To receive full credit, you must show your work!!!

1. In the following problems
   A. specify if the given equation is autonomous, linear and homogeneous, linear and nonhomogeneous, and/or separable, and
   B. find the its general solution.

(i) $\frac{\mathrm{d}y}{\mathrm{d}t} = 3 - 2y$

(ii) $\frac{\mathrm{d}y}{\mathrm{d}t} = (ty)^2$

(iii) $\frac{\mathrm{d}y}{\mathrm{d}t} = 3y + e^{7t}$

(iv) $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{ty}{1+t^2}$

(v) $\frac{\mathrm{d}y}{\mathrm{d}t} = 2ty^2 + 3y^2$

(vi) $\frac{\mathrm{d}y}{\mathrm{d}t} = t + \frac{2y}{1+t}$

(vii) $\frac{\mathrm{d}y}{\mathrm{d}t} = -5y + \sin 3t$

(viii) $\frac{\mathrm{d}y}{\mathrm{d}t} = -3y + e^{-2t} + 4$

(ix) $\frac{\mathrm{d}y}{\mathrm{d}t} = -y + t^2$

(x) $\frac{\mathrm{d}y}{\mathrm{d}t} = 2y - y^2$

(xi) $\frac{\mathrm{d}y}{\mathrm{d}t} = 3 + y^2$

(xii) $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{t^3 y}{1+t^4} + 2$

2. Solve the Initial Value Problem:

(i) $\frac{\mathrm{d}y}{\mathrm{d}t} = ty^2 + 2y^2; \quad y(0) = 1$

(ii) $\frac{\mathrm{d}y}{\mathrm{d}t} = -y^2; \quad y(0) = 0$

3. Use Euler's method with the given step size $\triangle t$ to approximate the solution to the given initial-vale problem over the time interval specified. Your answer should include a table of the approximate values of he dependent variable.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (3-y)(y+1), \quad y(0)=0, \quad 0 \le t \le 5, \quad \triangle t = 0.5$$

4. Find the equilibrium points of the system

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= 4x - 7y + 2 \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= 3x + 6y - 1\end{aligned}$$

5. Find the equilibrium points of the system

$$\begin{aligned}\frac{\mathrm{d}R}{\mathrm{d}t} &= 4R - 7F - 1 \\ \frac{\mathrm{d}F}{\mathrm{d}t} &= 3R + 6F - 12\end{aligned}$$

6. Convert the following second order differential equation into a first order system in terms of x and v, where $v = \mathrm{d}x/\mathrm{d}t$.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} - 3x + x^3 = 0$$

7. Rewrite the following linear system in
   A. Vector Form
   B. Component Form

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -3 & 2\pi \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

8. Rewrite the following linear system in
   A. Vector Form
   B. Component Form

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 0 & \beta \\ \lambda & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

9. A coefficient matrix for the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY}, \quad \text{where}\mathbf{Y}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

is specified. Also two functions and an initial value are given. For each system:

(a) Check that the two functions are solutions of the system; if they are not solutions then stop.

(b) Check that the two solutions are linearly independent; if they are not linearly independent then stop.

(c) Find the solution to the linear system with the given initial value.

$\mathbf{A} = \begin{pmatrix} -2 & -3 \\ 3 & -2 \end{pmatrix}$

Functions: $\mathbf{Y}_1(t) = (e^{-2t}\cos 3t, e^{-2t}\sin 3t)$; $\quad \mathbf{Y}_2(t) = (-e^{-2t}\sin 3t, e^{-2t}\cos 3t)$

Initial Value: $\mathbf{Y}(0) = (2, 3)$