$$\frac{dy}{dt} - 2y = 7e^{2t}; \quad y(0) = 3$$

(1) $\frac{dy}{dt} - 2y = 0$

$$\frac{dy}{dt} = 2y$$

$$\frac{dy}{2y} = dt$$

$$\frac{1}{2}\ln y = t + C_1$$

$$\ln y = 2t + C_2$$

$$e^{\ln y} = e^{2t + C_2}$$

$$\boxed{y_h = Ce^{2t}}$$

(2) $\frac{dy}{dt} - 2y = 7e^{2t}$

GUESS: $\alpha t e^{2t}$

$$\frac{d}{dt}\left(\alpha t e^{2t}\right) - 2\left(\alpha t e^{2t}\right) = 7e^{2t}$$

$$\alpha e^{2t} + \cancel{2\alpha t e^{2t}} - \cancel{2\alpha t e^{2t}} = 7e^{2t}$$

$$\alpha e^{2t} = 7e^{2t}$$

$$\alpha = 7$$

$$Y_p(t) = 7te^{2t}$$

$$Y(t) = Y_h(t) + Y_p(t)$$

$$\boxed{Y(t) = Ce^{2t} + 7te^{2t}}$$

Solve for C

$$3 = Ce^{2(0)} + 7 \cdot 0 \cdot e^{2 \cdot 0}$$

$$3 = Ce^{0} + 0$$

$$\boxed{3 = C}$$

$Y(t) = 3e^{2t} + 7te^{2t}$

Solution!

What is the general form of a homogeneous ODE?

$$\boxed{\frac{dy}{dt} = a(t)y}$$

What is the General Form of a nonhomogeneous ODE?

$$\boxed{\frac{dy}{dt} = a(t)y + b(t)}$$

What is an autonomous ODE?

$$\frac{dy}{dt} = f(y)$$

$\frac{dy}{dt} = 18y$ ← autonomous

$\frac{dy}{dt} = 18ty$ ← NOT!

EULER'S METHOD

$$t_{k+1} = t_k + \Delta t$$

$$y_{k+1} = y_k + f(t_k, y_k)\Delta t$$

where $f(t, y)$ is given by

$$\frac{dy}{dt} = \underline{f(t, y)}$$

INTEGRATING FACTORS

$\frac{dy}{dt} - \frac{2y}{t} = 2t^2 \; ; \; y(-2) = 4$ $\quad \frac{dy}{dt} + \boxed{(-)\frac{2y}{t}}$

$g(t) = -a(t)$

$$\frac{dy}{dt} = a(t)y + b(t)$$

$$\frac{dy}{dt} + g(t)y = b(t)$$

$$g(t) = -\frac{2}{t}$$

$$m(t) = e^{\int -\frac{2}{t} dt} = e^{-2\int \frac{1}{t} dt} = e^{-2\ln(t)}$$

$$= t^{-2} = \frac{1}{t^2}$$

$$\left[\frac{dy}{dt} + g(t)y = b(t)\right] m(t)$$

$$m(t)\frac{dy}{dt} + m(t)g(t)y = m(t)b(t)$$

$$m(t)\left[\frac{dy}{dt} - \frac{2}{t}y = 2t^2\right]$$

$$\frac{1}{t^2}\left[\; - \; - \; - \; - \; - \;\right]$$

$$\boxed{\frac{1}{t^2}\frac{dy}{dt} - \frac{2}{t^3}y = 2}$$

$\frac{d}{dt}(m(t)y)$ $\quad \frac{d}{dt}\left(\frac{1}{t^2}y\right) = 2$

$$\int d\left(\frac{1}{t^2}y\right) = \int 2\, dt$$

$$\frac{1}{t^2}y = 2t + C_1$$

$$\boxed{y = 2t^3 + C_1 t^2}$$

$y(-2) = 4$

$$4 = 2(-2)^3 + C_1(-2)^2$$

$$4 = -16 + 4C$$

$$20 = 4C \qquad C = 5$$

$y = 2t^3 + 5t^2$