HOMEWORK #5

(1i) $\frac{dy}{dt} = -\frac{y}{t} + 2$

$\frac{dy}{dt} + \frac{y}{t} = 2$

$g(t) = 1/t$

$\mu(t) = e^{\int g(t)dt} = e^{\int 1/t\, dt} = e^{\ln t}$

$\boxed{\mu(t) = t}$

$t \cdot \left[\frac{dy}{dt} + \frac{y}{t} = 2\right]$

$t\frac{dy}{dt} + y = 2t$

$\frac{d(ty)}{dt} = 2t$

$d(ty) = 2t\,dt$

$ty = 2\frac{t^2}{2} + C$

$y = t^2/t + C/t$

$\boxed{y = t + C/t}$

(1ii) $\frac{dy}{dt} = \frac{3}{t}y + t^5$

$\frac{dy}{dt} - \frac{3}{t}y = t^5$

$g(t) = -\frac{3}{t}$

$\mu(t) = e^{\int g(t)dt} = e^{\int -3/t\, dt} = e^{-3\ln t}$

$= e^{\ln t^{-3}} = t^{-3}$

$\boxed{\mu(t) = t^{-3}}$

$(t^{-3})\left[\frac{dy}{dt} - \frac{3}{t}y = t^5\right]$

$t^{-3}\frac{dy}{dt} - t^{-4}y = t^5 \cdot t^{-3}$

$\frac{d(t^{-3}y)}{dt} = t^2$

$d(t^{-3}y) = t^2\,dt$

$t^{-3}y = \frac{1}{3}t^3 + C$

$y = \frac{1}{3}t^6 + Ct^3$

$\boxed{y = \frac{t^6}{3} + Ct^3}$

(1iii) $\frac{dy}{dt} = -\frac{1}{1+t} + t^2$

$$\frac{dy}{dt} + \left(\frac{1}{1+t}\right)y = t^2$$

$$g(t) = \left(\frac{1}{1+t}\right)$$

$$\mu(t) = e^{\int g(t)\,dt} = e^{\int \frac{dt}{1+t}} = e^{\ln(1+t)}$$

$$\boxed{\mu(t) = 1+t}$$

$$(1+t)\left[\frac{dy}{dt} + \left(\frac{1}{1+t}\right)y = t^2\right]$$

$$(1+t)\frac{dy}{dt} + y = t^2 + t^3$$

$$\frac{d((1+t)y)}{dt} = t^2 + t^3$$

$$d((1+t)y) = (t^2 + t^3)\,dt$$

$$d((1+t)y) = t^2\,dt + t^3\,dt$$

$$(1+t)y = \frac{1}{3}t^3 + \frac{1}{4}t^4 + C$$

$$\boxed{y = \frac{t^4}{4(1+t)} + \frac{t^3}{3(1+t)} + \frac{C}{(1+t)} \quad \text{OR} \quad y = \frac{3t^4 + 4t^3 + C}{12(1+t)}}$$

(1iv) $\frac{dy}{dt} = -2ty + 4e^{-t^2}$

$$\frac{dy}{dt} + 2ty = 4e^{-t^2}$$

$$\boxed{g(t) = 2t}$$

$$\mu(t) = e^{\int 2t\,dt} = e^{2\frac{1}{2}t^2} = e^{t^2}$$

$$\boxed{\mu(t) = e^{t^2}}$$

$$e^{t^2}\left[\frac{dy}{dt} + 2ty = 4e^{-t^2}\right]$$

$$e^{t^2}\frac{dy}{dt} + 2te^{t^2}y = 4e^{-t^2}e^{t^2}$$

$$\frac{d(e^{t^2}y)}{dt} = 4$$

$$d(e^{t^2}y) = 4\,dt$$

$$e^{t^2}y = 4t + C$$

$$\boxed{y = 4te^{-t^2} + Ce^{-t^2}}$$

(1v) $\frac{dy}{dt} - \frac{2t}{1+t^2} y = 3$

$g(t) = -\frac{2t}{1+t^2}$

$\mu(t) = e^{\int g(t)} = e^{-\int \frac{2t}{1+t^2}} = e^{-\ln(1+t^2)} = e^{\ln(1+t^2)^{-1}}$

$$\boxed{\mu(t) = \frac{1}{1+t^2}}$$

$\left(\frac{1}{1+t^2}\right)\left[\frac{dy}{dt} - \frac{2t}{1+t^2} y = 3\right]$

$\left(\frac{1}{1+t^2}\right)\frac{dy}{dt} - \frac{2t}{(1+t^2)^2} y = \left(\frac{3}{1+t^2}\right)$

$\frac{d\left(\frac{y}{1+t^2}\right)}{dt} = \frac{3}{1+t^2}$

$d\left(\frac{y}{1+t^2}\right) = \frac{3\,dt}{1+t^2}$

$\frac{y}{1+t^2} = 3\arctan(t) + C$

$$\boxed{y = (1+t^2)\left[3\arctan(t) + C\right]}$$

(1vi) $\frac{dy}{dt} - \frac{2}{t} y = t^3 e^t$

3

$g(t) = -\frac{2}{t}$

$\mu(t) = e^{\int g(t)} = e^{\int -\frac{2}{t} dt} = e^{-2\ln(t)} = e^{\ln(t}$

$$\boxed{\mu(t) = t^{-2}}$$

$t^{-2}\left[\frac{dy}{dt} - \frac{2}{t} y = t^3 e^t\right]$

$t^{-2}\frac{dy}{dt} - \frac{2}{t^3} y = te^t$

$\frac{d(t^{-2}y)}{dt} = te^t$

$d(t^{-2}y) = te^t\,dt$

$t^{-2}y = te^t - e^t + C$

$t^{-2}y = (t-1)e^t + C$

$$\boxed{\begin{gathered} y = t^2(t-1)e^t + Ct^2 \\ \text{OR} \\ y = t^3e^t - t^2e^t + Ct^2 \end{gathered}}$$

(2i) $\frac{dy}{dt} = -\frac{1}{1+t} + 2$; $y(0) = 3$

$$\frac{dy}{dt} + \left(\frac{1}{1+t}\right)y = 2$$

$$g(t) = \left(\frac{1}{1+t}\right)$$

$$M(t) = e^{\int g(t)} = e^{\int \left(\frac{1}{1+t}\right)dt} = e^{\ln(1+t)}$$

$$\boxed{M(t) = 1+t}$$

$$(1+t)\left[\frac{dy}{dt} + \frac{1}{(1+t)}y = 2\right]$$

$$(1+t)\frac{dy}{dt} + y = 2(1+t)$$

$$\frac{d(y(1+t))}{dt} = 2(1+t)$$

$$d(y(1+t)) = 2(1+t)dt$$

$$(1+t)y = 2t + t^2 + C$$

$$y = \frac{2t}{1+t} + \frac{t^2}{1+t} + \frac{C}{1+t}$$

$$y = \frac{2t + t^2 + C}{1+t}$$

$$\boxed{y = \frac{t^2 + 2t + C}{t+1}}$$

$y(0) = 3$

$$3 = \frac{0^2 + 2(0) + C}{0+1}$$

$$3 = \frac{C}{1}$$

$$\boxed{3 = C}$$

$$\boxed{y = \frac{t^2 + 2t + 3}{t+1}}$$

(2ii) $\frac{dy}{dt} = \frac{1}{t+1}y + 4t^2 + 4t$; $y(1) = 10$

$$\frac{dy}{dt} - \frac{1}{t+1}y = 4t^2 + 4t$$

$$\boxed{g(t) = -\frac{1}{t+1}}$$

$$M(t) = e^{-\int \frac{1}{t+1}dt} = e^{-\ln(1+t)} = e^{\ln(t+1)^{-1}}$$

$$M(t) = \left(\frac{1}{1+t}\right)$$

$$\left(\frac{1}{1+t}\right)\left[\frac{dy}{dt} - \frac{1}{1+t}y = 4t^2 + 4t\right]$$

$$\left(\frac{1}{1+t}\right)\frac{dy}{dt} - \frac{1}{(1+t)^2}y = \frac{4t(t+1)}{t+1}$$

$$\frac{d\left(\frac{1}{1+t}y\right)}{dt} = 4t$$

$$d\left(\frac{y}{1+t}\right) = 4t\,dt$$

$$\frac{y}{1+t} = 2t^2 + C$$

$$y = 2t^2(1+t) + C(1+t)$$

$$y = (2t^2 + C)(1+t)$$

$$\boxed{y = 2t^3 + 2t^2 + Ct + C}$$

$y(1) = 10$

$$10 = 2(1)^3 + 2(1)^2 + C(1) + C$$

$$10 = 2 \cdot 1 + 2 \cdot 1 + C + C$$

$$10 = 2 + 2 + 2c$$

$$6 = 2c$$

$$\boxed{3 = c}$$

$$\boxed{y = 2t^3 + 2t^2 + 3t + 3}$$

(2iii) $\frac{dy}{dt} = -\frac{y}{t} + 2; y(1) = 3$

$$\frac{dy}{dt} + \frac{y}{t} = 2$$

$$g(t) = \frac{1}{t}$$

$$\mu(t) = e^{\int g(t)} = e^{\int \frac{1}{t} dt} = e^{\ln(t)}$$

$$\boxed{\mu(t) = t}$$

$$t\left[\frac{dy}{dt} + \frac{1}{t}y = 2\right]$$

$$t\frac{dy}{dt} + y = 2t$$

$$\frac{d(ty)}{dt} = 2t$$

$$d(ty) = 2t\,dt$$

$$ty = t^2 + C$$

$$y = \frac{t^2}{t} + \frac{C}{t}$$

$$\boxed{y = t + \frac{C}{t}}$$

$$y(1) = 3$$

$$3 = 1 + \frac{C}{1}$$

$$3 = 1 + C$$

$$\boxed{2 = C}$$

$$\boxed{y = t + \frac{2}{t}}$$

We also solved this in problem #1ii!

(2iv) $\frac{dy}{dt} = -2ty + 4e^{-t^2}; y(0) = 3$

We solved this problem in problem (#1iv)

$$\boxed{y = 4te^{-t^2} + ce^{-t^2}}$$

$$3 = 4(0)e^{0^2} + ce^{0}$$

$$3 = 0 + c$$

$$\boxed{3 = c}$$

$$\boxed{y = 4te^{-t^2} + 3e^{-t^2}}$$

(2 v) $\frac{dy}{dt} - \frac{2}{t}y = 2t^2$; $y(-2)=4$

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$g(t) = -\frac{2}{t}$

$\mu(t) = e^{\int g(t)} = e^{\int -\frac{2}{t}dt} = e^{-2\int \frac{dt}{t}} = e^{-2\ln(t)} = e^{\ln(t)^{-2}}$

$\boxed{\mu(t) = t^{-2}}$

$t^{-2}\left[\frac{dy}{dt} - \frac{2}{t}y = 2t^2\right]$

$t^{-2}\frac{dy}{dt} - \frac{2}{t^3}y = 2$

$\frac{d(t^{-2}y)}{dt} = 2$

$d(t^{-2}y) = 2dt$

$t^{-2}y = 2t + c$

$\boxed{y = 2t^3 + ct^2}$

$y(-2) = 4$

$4 = 2(-2)^3 + c(-2)^2$

$4 = 2(-8) + 4c$

$4 = -16 + 4c$

$20 = 4c$

$\boxed{5 = c}$

$\boxed{y = 2t^3 + 5t^2}$

(2 vi)

I wrote the problem wrong and as such, there is no analytic solution. Therefore, this is a freebie question as long as you attempted it!