DUE: Never!

This is for your own practice!!!
We will go over these on Thursday!!!

1. In the following exercises, a coefficient matrix for the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY}, \quad \text{where} \mathbf{Y}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

is specified. Also two functions and an initial value are given. For each system:

(a) Check that the two functions are solutions of the system; if they are not solutions then stop.

(b) Check that the two solutions are linearly independent; if they are not linearly independent then stop.

(c) Find the solution to the linear system with the given initial value.

(i) $\mathbf{A} = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix}$
Functions: $\mathbf{Y}_1(t) = (e^{-3t}, e^{-3t})$; $\quad \mathbf{Y}_2(t) = (e^{-4t}, e^{-4t})$
Initial Value: $\mathbf{Y}(0) = (2, 3)$

(ii) $\mathbf{A} = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix}$
Functions: $\mathbf{Y}_1(t) = (e^{-3t} - 2e^{-4t}, e^{-3t} - 4e^{-4t})$; $\quad \mathbf{Y}_2(t) = (2e^{-3t} + e^{-4t}, 2e^{-3t} + e^{-4t})$
Initial Value: $\mathbf{Y}(0) = (2, 3)$

(iii) $\mathbf{A} = \begin{pmatrix} -2 & -3 \\ 3 & -2 \end{pmatrix}$
Functions: $\mathbf{Y}_1(t) = (e^{-2t} \cos 3t, e^{-2t} \sin 3t)$; $\quad \mathbf{Y}_2(t) = (-e^{-2t} \sin 3t, e^{-2t} \cos 3t)$
Initial Value: $\mathbf{Y}(0) = (2, 3)$

(iv) $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$
Functions: $\mathbf{Y}_1(t) = (-e^{-t} + 12e^{3t}, -e^{-t} + 4e^{3t})$; $\quad \mathbf{Y}_2(t) = (-e^{t}, 2e^{-t})$
Initial Value: $\mathbf{Y}(0) = (2, 3)$